

## The Performance Gradient System ©

by Bill Grauer

In scoring for a long series there is always the problem of how to account for the situation in which one race of the series has a different number of racers than another race. This often happens in a long series. Different systems handle the problem differently. In the US Sailing, Low-Point System and with rule A9, each race is considered to have the same number of boats – the number equal to taking all boats that raced in any of the races. In another system, the Cox-Sprague system, the boats are assigned a different number of points, for the same finish position, based on the number of boats in that race. US Sailing also discusses other scoring systems for long series, such as the High-Point Percentage and the Low-Point Average system (see US Sailing article entitled, *Alternative Scoring Systems for Long Series*). These systems suffer from procedural complexities and have similar problems. These problems can be illustrated by examining the Cox-Sprague system.

In the past several years the SBCC has used the Cox-Sprague system. This system has some oddities that need to be understood. A portion of the table used for the Cox-Sprague system is shown below. The first row shows the number of boats that started in a particular race. The first column shows the boats finish position. The numbers in parenthesis shows the number of points given for boats that did not race, retired, or were disqualified.

	2	3	4	5	6	7	8	9	10	11
Finish										
1	10	31	43	52	60	66	72	76	80	84
2	4	25	37	46	54	60	66	70	74	78
3	0	21	33	42	50	56	62	66	70	74
4		(17)	29	38	46	52	58	62	66	70
5			(26)	35	43	49	55	59	63	67
6				(32)	40	46	52	56	60	64
7					(38)	44	50	54	58	62
8						(42)	48	52	56	60
9							(46)	50	54	58
10								(48)	52	56

If we look at a race that had five starters, the first place boat gets 52 points. If we look at the race that has eight starters, the sixth place boat also gets 52 points. So, first place and sixth place get the same number of points. This does not seem equitable when only three boats are added to the fleet.

In the past, the SBCC has used the above Cox-Sprague table for scoring the season Class Championship Series. Another method of utilizing the Cox-Sprague system can be employed to come up with a normalized score. This is done by taking the points awarded

and dividing it by the maximum number of points. Thus, in the example given, the first place boat would get a score of 1 (52/52) in the five boat race, and the sixth place boat would get a score of 0.722 (52/72) in an eight boat fleet. This normalized technique (not used by SBCC) corrects the problem cited in the example. But this normalization introduces its own problem. The problem is that the winner always gets a normalized score of 1. We all know that a boat that finished first in a fleet of ten starters has done better than a boat that finished first in a fleet of three starters, so why should they both get the same score of 1?

To solve these problems I have developed the system that I will now describe. This new system gives a first place boat a higher score the more boats that are in the race. The logic that I used to derive this system is based on the following observation. For a boat that finishes first in a two boat race has finished in the upper half of the fleet. That's all we can say. So perhaps we should give that boat a score between 50% and 100%. As a first order approximation of a solution, let us split the difference between 50% and 100% and give the boat a score of 75%. Now, in a ten boat race we can say that the winner was in the top 10% of the fleet, and so, should get a score somewhere between 90% and 100%. Again, let us split the difference and give that boat a score of 95%. This now awards the first place boat in a large fleet a better number than the first place boat in a two boat fleet.

Now we must consider how to handle boats that are disqualified, did not start, retired, etc. To handle this I have introduced the concept of the phantom racer who always comes in last place. All the boats that are disqualified, did not start, retired, etc., are assigned to this phantom racer position. Then I use the same system to calculate a new score. For the first place boat in a two boat race, I envisage a three boat race – the two starters and the phantom boat. The first place boat is therefore in the top third of the fleet and should get a score between 66% and 100%. Splitting the difference, this boat would get 83%. The first place boat in a ten boat fleet would be scored as though the fleet consisted of eleven boats – one more for the phantom racer. Using the same methodology, the score would be half way between 90.91% and 100%, or 95.5%.

The scores are changed from a percentage to a decimal and the result is called a 'Performance Gradient', or simply '**G**'. The **G** can be calculated with a simple formula that can be put into a cell of a spreadsheet. The formula is:

$$G = 1 - [(p - 1)/(n + 1)] - [1/\{2(n + 1)\}];$$

Where **p** is the boat's finish position (1 for first place, 2 for second place, etc.), and **n** is the number of actual starters in the race, not counting the phantom racer. Boats that are disqualified, did not start, retired, etc., get a **p** equal to **n** + 1 (for the phantom racer).

The **G** will be a number less than 1 but greater than 0. The table below shows the **G** for races with up to ten boats starting.

Table of Performance Gradients calculated with the formula;  $G = 1 - [(p - 1)/(n + 1)] - [1/\{2(n + 1)\}]$

Number of Boats in the Fleet (Down)	Finish Position (Across)										
	1	2	3	4	5	6	7	8	9	10	11
2	0.833	0.5	0.1667								
3	0.875	0.625	0.375	0.125							
4	0.9	0.7	0.5	0.3	0.1						
5	0.917	0.75	0.583	0.417	0.25	0.0833					
6	0.929	0.786	0.643	0.5	0.357	0.214	0.0714				
7	0.937	0.812	0.687	0.562	0.437	0.312	0.187	0.0625			
8	0.944	0.833	0.722	0.611	0.5	0.389	0.278	0.167	0.0555		
9	0.95	0.85	0.75	0.65	0.55	0.45	0.35	0.25	0.15	0.05	
10	0.955	0.864	0.773	0.682	0.591	0.5	0.409	0.318	0.227	0.136	0.04545

The **G** score for the phantom racer is shown as the last number in each row. As you can see, the first place boat in a fleet of five boats gets a **G** of 0.917 and the sixth place boat in a fleet of eight boats gets a **G** of 0.389 which seems to be a reasonable spread. Also, note that the second place boat in a fleet of three boats gets a score of 0.625 and the fourth place boat in a fleet of eight boats gets 0.611. This also seems about right because in both cases these boats finished near the middle of the fleet.

In some cases it may be desirable to express the performance as a percent instead of a decimal. For this, I introduce the 'Performance Number', **N** which is just 100 times **G**. So, for the Performance Number, we would have:  $N = 100 * G$ .

With the Performance Gradient or Performance Number, we have a simple system that avoids all the problems seen in the Cox-Sprague system. In addition, this system is simpler to implement on a computer because it is based on a formula rather than having to utilize (and program) a table. The formula can be placed in a cell of a spreadsheet program for example.